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Structure functions of turbulence in the atmospheric boundary layer over the ocean

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Structure functions of turbulent velocity fluctuations up to fourth order have been measured at several heights in the atmospheric boundary layer over the open ocean, and the results are compared with theoretical predictions for separations in the inertial subrange. The behaviour of second- and third-order quantities shows substantial agreement with the predictions of Kolmogorov's original theory over a wide range of separations, but the results of a recent modification of the theory, attempting to account for intermittency in the local dissipation rate, are also consistent with the data over somewhat shorter separation intervals. The behaviour of the measured fourth-order structure function disagrees with that predicted from Kolmogorov's original work, but good agreement is found with the results of the modified theory.

1. Introduction

For turbulent flows at sufficiently high Reynolds numbers, the original theory of local isotropy of Kolmogorov (1941 *a*, *b*, *c*), and later extensions by Kolmogorov (1962), Obukhov (1962), and Yaglom (1966), furnish a number of conflicting predictions for the behaviour of structure functions, correlations, and spectra of velocity fluctuations in the inertial subrange. Although several workers have reported the results of investigations to test the degree of local isotropy in spectra and the behaviour of corresponding second-order structure functions, measurements of the higher-order quantities, which are generally more sensitive to recent theoretical refinements, are relatively scarce. Obukhov (1951) made atmospheric measurements of the second-order structure function

$$D_{nn}(r) = \tfrac{1}{2}\langle (u-u')^2\rangle,$$

where u and u' were the longitudinal velocities measured at two points separated laterally by a distance r, and found it increased like $r^{\frac{2}{3}}$ in the inertial subrange as originally predicted by Kolmogorov (1941 *a*). Grant, Stewart & Molliet (1962) and Pond, Stewart & Burling (1963), measured energy spectra in a tidal channel and in the wind over waves respectively. They found the corresponding result that the one-dimensional energy spectrum $\phi(k_1)$ was consistent with the relation

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$\phi(k_1) = K'\epsilon^{\frac{2}{3}}k_1^{-\frac{5}{3}}$ over a considerable range, where $k_1 = 2\pi f/U$, f is frequency (Hz), U is the mean velocity, ϵ is the mean rate of dissipation per unit mass, and

$$\langle u^2 \rangle = \int_0^\infty \phi(k_1)\,dk_1.$$

The only reported measurements of higher-order moments obtained in a turbulent flow exhibiting an inertial subrange in the energy spectrum appear to be the atmospheric boundary-layer measurements of Gurvich (1960) and Stewart, Wilson & Burling (1970). Gurvich measured the skewness factor for two different spatial separations, but made no systematic attempt to determine the general behaviour of the skewness factor or triple correlation function with varying separation distance. Some data concerning the variation of skewness and flatness factors with separation distance were discussed by Stewart *et al.* (1970) but no measurements of the individual structure functions were presented. Measurements of triple correlations and skewness factors in locally isotropic laboratory grid turbulence have been reported by Townsend (1948), Stewart (1951), Frenkiel & Klebanoff (1967), and Van Atta & Chen (1968), but these data were all obtained at Reynolds numbers too low for the existence of an inertial subrange. The present measurements in the turbulent boundary layer over the ocean are the results of an initial attempt to study the behaviour of higher-order moments of the fluctuating velocity in the inertial subrange for comparison with existing theoretical predictions.

2. Theoretical relations

The nth-order structure function is $\langle (u-u')^n \rangle$, where r is the separation distance of two points, and u and u' are the components of the fluctuating velocity at the two points in the direction of the separation. For separations in the inertial subrange, $\eta \ll r \ll L$, where $\eta = (\nu^3/\epsilon)^{\frac{1}{4}}$ is the Kolmogorov length scale, ν is the kinematic viscosity, ϵ is the mean rate of dissipation of kinetic energy, and L is the characteristic length scale of the energy containing eddies. According to Kolmogorov's original theory, in the inertial subrange the structure functions depend only on r and ϵ, and hence by dimensional analysis

$$\langle (u-u')^n \rangle = C_n(\epsilon r)^{\frac{1}{3}n}, \tag{1}$$

where the C_n are universal constants. The form of the third-order structure function in the inertial subrange can also be derived directly from the Kármán–Howarth equation and the constant C_3 is thereby evaluated *a priori*. Thus, for n equal to 2, 3, and 4 we have

$$\left.\begin{aligned} D_{dd}(r) &= \langle (u-u')^2 \rangle = C_2(\epsilon r)^{\frac{2}{3}}, \\ B_{ddd}(r) &= \langle (u-u')^3 \rangle = -\tfrac{4}{5}\epsilon r, \\ B_{dddd}(r) &= \langle (u-u')^4 \rangle = C_4(\epsilon r)^{\frac{4}{3}}. \end{aligned}\right\} \tag{2a, b, c}$$

Since (2b) contains no unknown constants that must be determined empirically, a direct absolute comparison may be made with experimental data.

When (2*b*) is made dimensionless, using the Kolmogorov length scale and the Kolmogorov velocity scale $v_k = (\nu\epsilon)^{\frac{1}{4}}$, it becomes

$$B_{ddd}(r)/v_k^3 = -0{\cdot}8r/\eta. \tag{3}$$

A further consequence of (2) is that the skewness factor

$$S = \langle(u-u')^3\rangle/\langle(u-u')^2\rangle^{\frac{3}{2}}$$

and flatness factor

$$F = \langle(u-u')^4\rangle/\langle(u-u')^2\rangle^2$$

must be constant for $\eta \ll r \ll L$. As discussed in detail by Hinze (1959) and Pond, Stewart & Burling (1963), the constants C_2, K', and S are simply related, if spectral contributions to D_{dd} and B_{ddd} from outside the inertial subrange are negligible. One expects this condition to be nearly satisfied if the Reynolds number of the turbulence is large. The magnitude of the skewness factor is then related to K' and C_2 by the relations,

$$S = -0{\cdot}1(K')^{-\frac{3}{2}} = -\tfrac{4}{5}C_2^{-\frac{3}{2}}. \tag{4}$$

According to Landau & Lifshitz (1959), the arguments leading to the preceding relations do not take into account the possible influence of the statistical distribution of fluctuations in the local dissipation rate $\tilde{\epsilon}(\bar{x},t)-\epsilon$. The pronounced intermittency of the smaller scales of atmospheric turbulence produces a large dispersion in $\tilde{\epsilon}$, and according to the model of the cascade process described by Yaglom (1966) the variance of $\log\tilde{\epsilon}_r$ ($\tilde{\epsilon}_r$ is the average of $\tilde{\epsilon}$ over a sphere of diameter r) is given by the expression,

$$\sigma^2_{\log\tilde{\epsilon}_r} = A(\bar{x},t) + \mu\log L/r,$$

where μ is a universal constant and $A(\bar{x},t)$ depends on the macrostructure of the flow. In addition, the probability distribution of $\tilde{\epsilon}$ is found to be lognormal. The spectrum of the fluctuations in local dissipation is given by

$$E_{\tilde{\epsilon}\tilde{\epsilon}}(k) \sim k^{-1+\mu}, \tag{5}$$

and (1) is replaced by a more general expression of Kolmogorov (1962):

$$\langle(u-u')^n\rangle = \tilde{C}_n(\bar{x},t)\,(\epsilon r)^{\frac{1}{3}n}\,(L/r)^{\mu n(n-3)/18}, \tag{6}$$

where the $\tilde{C}_n(\bar{x},t)$ are now not absolute constants, but may depend on the macrostructure of the flow. These studies thus predict an increase in the exponent of r in the relation for the second-order structure function, and a corresponding decrease in the exponent of k in the energy spectrum. The third-order structure function remains linear in r, as required by (2*b*), which is an independent consequence of the Navier–Stokes equation. The exponent of r is decreased for higher-order structure functions.

The modified expressions for the skewness and flatness factors are

$$S(r) = -\frac{4}{5}\frac{1}{(\tilde{C}_2)^{\frac{3}{2}}}(L/r)^{\frac{1}{6}\mu}, \tag{7}$$

$$F(r) = \frac{\tilde{C}_4}{\tilde{C}_2^2}\left(\frac{L}{r}\right)^{\frac{4}{9}\mu}.$$

3. Experimental arrangement and data analysis

The data were obtained in the period 17–18 May 1969, in fair trade wind conditions during the Barbados Oceanographic and Meteorological Experiment. The measurements were made from FLIP, the stable floating instrument platform of the Scripps Institution of Oceanography, at a location of 13-00N55-OOW, about 400 km east of the island of Barbados. The local sea state consisted of approximately 1·5 m swells with 30–70 cm wind waves.

The fluctuating component of the turbulent velocity field in the mean wind direction u was measured at three heights above mean sea level with a single vertically oriented hot-wire 5μ in diameter and 1 mm long. The hot-wire was operated in the constant resistance mode using a DISA 55D05 anemometer, and the anemometer output was linearized with a DISA 55D10 linearizer. At the lowest level, a boom arrangement placed the hot-wire probe about 15 m to one side of FLIP's hull to minimize flow interference, while for the upper levels the probe was mounted on top of a telescoping tower, which could be extended a maximum of about 20 m above the upper deck.

The hot-wire was normally calibrated with a cup anemometer at the same vertical level, using 100 second averaged readings of the two instruments. Some hot-wire calibrations were also carried out inside the lab aboard FLIP using a small calibration wind-tunnel.

The relatively slow convergence of measurements of third-order moments (recognized by Gurvich, whose longest sample of data lasted 24 min), a lack of *a priori* theoretical estimates of the expected rate of convergence for odd moments, and experience with similar measurements in the laboratory made it desirable to obtain the longest possible continuous records of data. The linearized hot-wire signal was therefore FM tape recorded at a speed of 19·1 cm/sec, providing the longest possible uninterrupted runs (about 80 min of data plus calibrations) while preserving adequate frequency response over the entire range of the turbulent spectrum. The analog tape was later played back and sampled with a twelve bit analog to digital converter at a rate of 521·5 samples/sec. The signal was d.c. coupled throughout the entire recording-playback system to avoid any possibility of errors in the measured triple correlations due to high-pass filtering as described by Van Atta & Chen (1968). For various reasons, the longest series of data used in the present computations corresponded to about 66 min (at 3 m), while the runs for 21 and 31 m were about 53 and 26 min, respectively.

To obtain some minimal measure of the variability of the measurements, the digital data for each height were first processed in shorter sections each consisting of 409,600 samples. Velocity differences were computed using every fourth sample as the initial point. The structure functions, skewness factor, and flatness factor were computed for each section; these results were then averaged over all the available data for each height, to produce the final measured values. Taylor's hypothesis in the form $r = Ut$ was used to interpret the measured time delay as a spatial separation r.

The mean rate of dissipation of energy ϵ was estimated in the usual way by measuring the mean square value of the time derivative (obtained with a differentiating circuit) of the velocity $\langle(\partial u/\partial t)^2\rangle$, invoking Taylor's hypothesis in the form

$$\frac{\partial}{\partial x} = -\frac{1}{U}\frac{\partial}{\partial t},$$

and using the isotropic relation $\epsilon = -15\nu\langle(\partial u/\partial x)^2\rangle$ to compute ϵ.

To assess the influence of fluctuations in $\tilde{\epsilon}$, the value of μ was estimated using the digitized velocity derivative data to compute $E_{\epsilon_1\epsilon_1}$, the energy spectrum of $\epsilon_1 = (\partial u/\partial x)^2$, as in earlier work by Gurvich & Zubkhovski (1963) and Pond & Stewart (1965).

4. Results and discussion

4.1. *Second-order structure functions*

The second-order structure function is needed to compute the skewness factor S, and its behaviour also serves as one criterion for the existence and extent of inertial subrange behaviour which may be compared with earlier measurements and theoretical predictions. The measured second-order structure functions are shown in figure 1. The symbols used here to identify data obtained for different vertical heights, z, denote the same data and levels throughout the following figures. The data for each individual height are fairly well fitted by Kolmogorov's original relation with $D_{dd} \sim r^{\frac{2}{3}}$ over a considerable range for separations larger than about 5 cm. The data for the higher levels fit the power law expression best, whereas the $z = 3$ m data exhibit a distinct curvature. It is possible that the $z = 3$ m data were influenced in some measure by the wind waves and swell, but the present measurements allow no assessment of this influence. For $r < 5$ cm, the data deviate from this relation, as expected in the viscous range where D_{dd} should vary like r^2. For the 31 m level, the data follow the inertial subrange relation up to the largest separations considered (about 44 m), considerably extending the result of Obukhov, who measured $D_{nn}(r)$ using two anemometers separated at most by 60 cm. Corresponding energy spectra computed for the highest level also exhibited inertial subrange-like behaviour ($E \sim k^{-\frac{5}{3}}$) extending to these large (anisotropic) scales as noted by previous investigators. The 3 m and 21 m data exhibit a departure from $D_{dd} \sim r^{\frac{2}{3}}$ behaviour for all separations larger than about twice the height of the probe above mean sea level. According to Pond *et al.* (1963), a reasonable necessary condition for local isotropy is given by $kz \gg 4{\cdot}5$, where z is the height above mean sea level. Taking

$$k = 2\pi f/U = 2\pi/\lambda = 2\pi/r,$$

the present evidence suggests that the region of local isotropy might extend to all length scales for which $kz \geqslant \pi$. As found by previous investigators, the $E \sim k^{-\frac{5}{3}}$ behaviour of the corresponding energy spectra for $z = 3$ m extended to the largest scales for which the spectra were computed (about 40 m). Thus, it appears that deviations in the behaviour of the second-order structure function from inertial subrange form may be a considerably more accurate and sensitive criterion for local isotropy than spectral measurements, since the latter follow the inertial subrange law for scales so large that the flow cannot be locally isotropic. Although the spectra and correlation functions are of course Fourier transforms of one another, there is no *a priori* exact correspondence between the extent of the power law inertial subranges for intermediate values of the variables r and k, so that one obtains more information and insight concerning the structure of the turbulence by computing both of them.

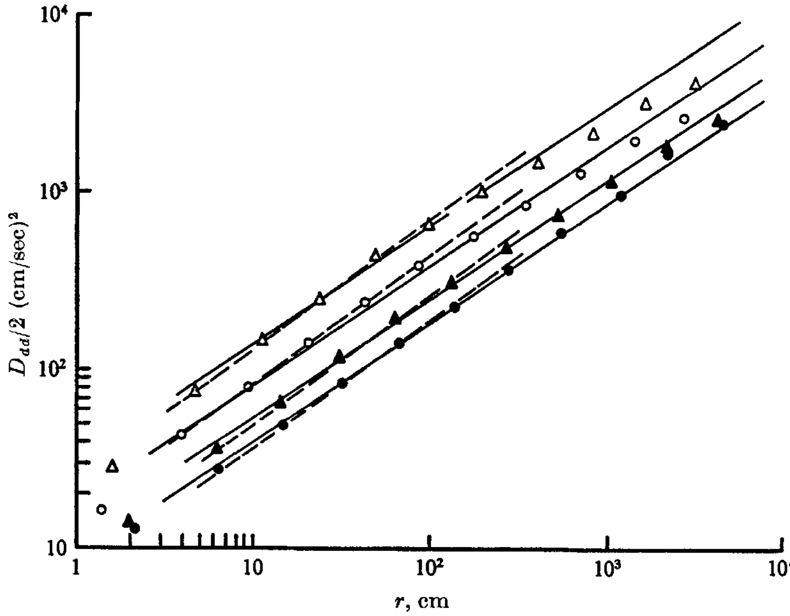

FIGURE 1. Second-order structure functions. Here, and in figures 3–7, the meaning of the symbols is as follows: ○, z = 3 m, U = 7·2 m/s; △, z = 3 m, U = 8·4 m/s; ▲, z = 23 m, U = 11 m/s; ●, z = 31 m, U = 11·3 m/s. The solid lines have a slope of $\frac{2}{3}$. - - - -, a slope of 0·722.

In figure 2, the measured $D_{dd}(r)$, normalized with the Kolmogorov velocity scale v_K, are plotted *versus* the separation distance normalized with the Kolmogorov length scale η. The Kolmogorov length scale is about 0·8 mm for the $z = 3$ m data and 1·1 mm for $z = 21$ m and $z = 30$ m. The data for the three heights collapse fairly well into a single curve. The apparent lower limit of the inertial subrange is at about 50η, which is a factor of two or three larger than is usually inferred from energy spectra. The average value of C_2 determined from fits to the data is 2·3, or, using (4), $K' = 0{\cdot}58$. Corresponding values of K' determined directly from the energy spectra were systematically higher than those obtained from C_2, with an average value of 0·70, indicating that spectral contributions to D_{dd} outside the inertial subrange are not negligible. All values of K' determined from C_2 and one of the values determined from the spectra lie within the scatter of previous spectral measurements compiled by Pond *et al.* (1963). The remaining values determined from the spectra appear to be unusually large, although similarly large values in the range 0·60–0·69 have been reported by Gibson *et al.* (1970) from earlier FLIP measurements during BOMEX.

The second-order structure function data can be reinterpreted according to the refined theory of inertial subrange similarity taking into account the variations in $\tilde{\epsilon}$. An example of the data and procedure used for estimating μ directly from the energy spectra $E_{\epsilon_1\epsilon_1}$ of $(\partial u/\partial t)^2$ is shown in figure 3. The average measured slope of these spectra in the inertial subrange was very nearly -0.5, giving

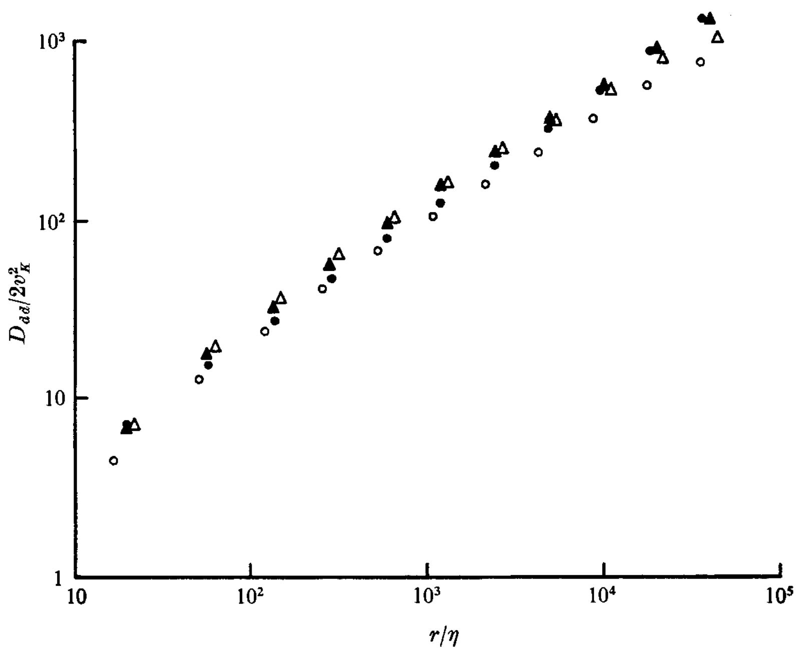

FIGURE 2. Second-order structure functions normalized with Kolmogorov scales.

$\mu = 0.5$, a value close to the mean of the scatter in earlier measurements cited previously. Then, according to (6), the power of r in $D_{dd}(r)$ increases by 0·056, so that $D_{dd}(r) \sim r^{0.722}$. As shown in figure 1, this expression fits the data for $5\,\text{cm} \leqslant r \leqslant 80\,\text{cm}$ very well for all z, and in fact produces a considerably better fit than $r^{\frac{2}{3}}$ in this range. This range is considerably smaller than that for which the $r^{\frac{2}{3}}$ expression provides a fairly good overall fit to the data. The maximum separation for which $D_{dd}(r) \sim r^{0.722}$ appears to be rather insensitive to variations in z. For $z = 3\,\text{m}$, the value of this maximum separation predicted by the expression $kz = 4.5$ is of the same order as the observed value (4·2 m as compared with about 1 m observed), but for $z = 21\,\text{m}$ and 31 m the predicted value of 43·2 m is much larger than observed. From figure 2, $r/\eta \simeq 10^3$ is the maximum length scale for which the turbulence can be locally isotropic according to the refined criterion for $D_{dd}(r)$. This result is in qualitative agreement with measurements of the extent of local isotropy in the boundary layer over the sea by Weiler & Burling (1967). Measuring the ratio of the transverse and longitudinal energy spectra, they found that as the length scale decreased this ratio increased slowly from zero for large scales to only about 15 % of the value for isotropic turbulence at length scales of order $r/\eta = 10^3$. The ratio increased most rapidly for $10^3 \geqslant r/\eta \geqslant 10^2$ and continued to increase at a decreasing rate for smaller scales, although never reaching the isotropic value even for the smallest scales considered ($r/\eta \simeq 10$). These measurements and the present ones of $D_{dd}(r)$ thus indicate the same restricted range of local isotropy, and the more extensive $r^{\frac{2}{3}}$ range in D_{dd} could be interpreted as accidental in the same sense as the persistent $k^{-\frac{5}{3}}$ behaviour of the energy spectrum for very small wave-numbers. These remarks, which are based on very limited data, do not rule out the alternate

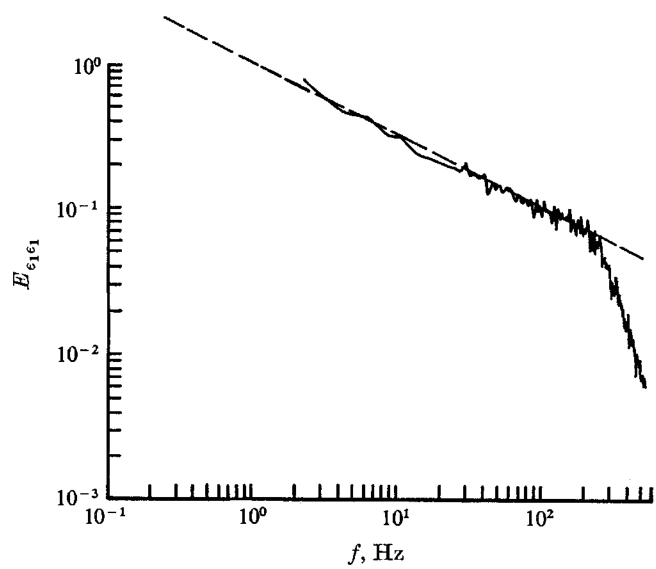

FIGURE 3. Energy spectrum of $(\partial u/\partial x)^2$ for z = 31 m, U = 11·3 m/s. The dashed line has a slope of $-0{\cdot}5$.

possibility that the agreement of the data with the modified expression for D_{dd} in the range $5_{\text{cm}} \leqslant r \leqslant 80_{\text{cm}}$ may itself be fortuitous. A fit over some restricted range is to be expected, since the slope must eventually increase as r decreases toward the viscous subrange where $D_{dd} \sim r^2$. Because of this relative loss of variance at small r due to viscosity, one might expect that on a linear plot power law fits to the measured structure fractions would not pass through zero, but through some small negative value. This effect was found to be negligibly small for the present data, as expected for high Reynolds number turbulence.

4.2. *Third-order structure functions*

The scatter of the measured third-order structure functions for sub-sections of data consisting of 409,600 samples each was much larger than for the corresponding measurements of the second-order structure function. The differences between individual runs increased considerably as the separation distance increased. Even for the longest runs, it appears that the duration of the records obtained in the present measurements may be marginally adequate to produce statistical convergence in computations of third-order quantities.

The final measured third-order structure functions, normalized with Kolmogorov velocity and length scales are shown in figure 4. As the separation distance increases within the inertial subrange, the third-order structure function increases monotonically, and, for $r/\eta < 10^3$, the data collapse fairly well into a single curve, and cluster around the theoretical prediction (equation 3) of Kolmogorov. In contrast with measurements of energy spectra and second-order

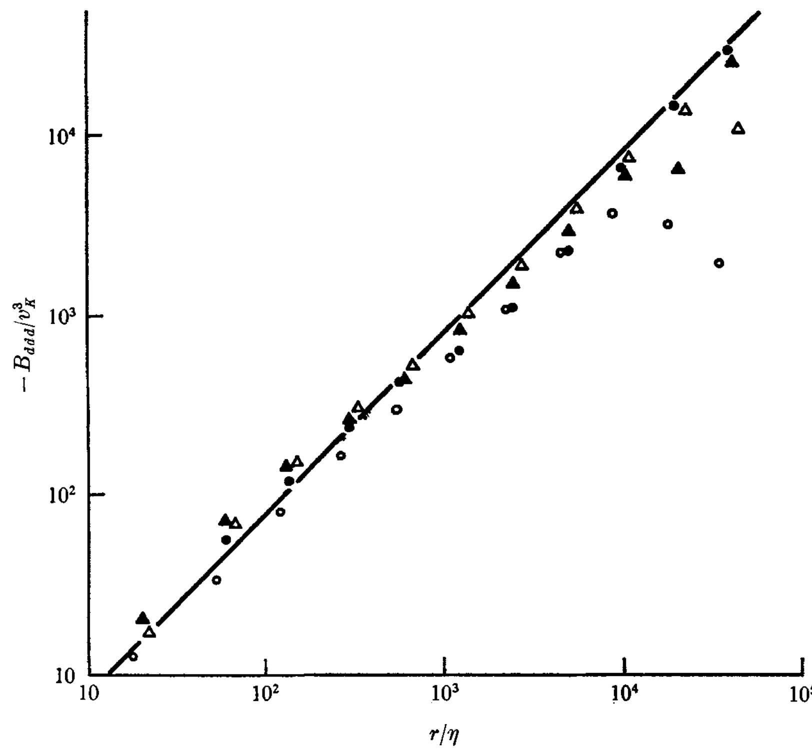

FIGURE 4. Third-order structure functions normalized with Kolmogorov scales. The solid line is the theoretical prediction from Kolmogorov's inertial subrange analysis of the Kármán–Howarth equation.

correlations, this comparison of theory and experiment for third-order quantities is absolute, involving no unknown (universal or otherwise) constants. The observed approximately linear behaviour and absolute agreement lends further support to Kolmogorov's theory of local isotropy. From the present limited amount of data, it appears that the linear dependence of the structure function on separation distance may extend to larger distances as the height above the surface is increased. Generally, this upper separation limit is undoubtedly a function of the local geometry of the large-scale motion in the particular flow under study.

The measured skewness factors are shown in figure 5. The data for all z collapse fairly well in the range $10^2 \leqslant r/\eta \leqslant 10^4$. The skewness factor decreases very slowly with increasing separation distance, decreasing from 0·4 to 0·2 as the separation varies by a factor of 10^3. This behaviour is qualitatively similar to that reported by Stewart *et al.* (1970). For each individual run at a given z, S appears to reach a plateau in the range $3\times 10^2 \leqslant r/\eta \leqslant 10^4$, where $D_{ddd}(r)$ and $D_{dd}(r)$ vary nearly like r and $r^{\frac{2}{3}}$, respectively, as predicted by Kolmogorov. From (7), the effect of the fluctuations in $\bar{\epsilon}$ would be to cause S to decrease very slowly (like $r^{-0.083}$) in this range. The present data are obviously not sufficiently precise to distinguish between $S =$ constant and $S \sim r^{-0.08}$. The mean measured value of S

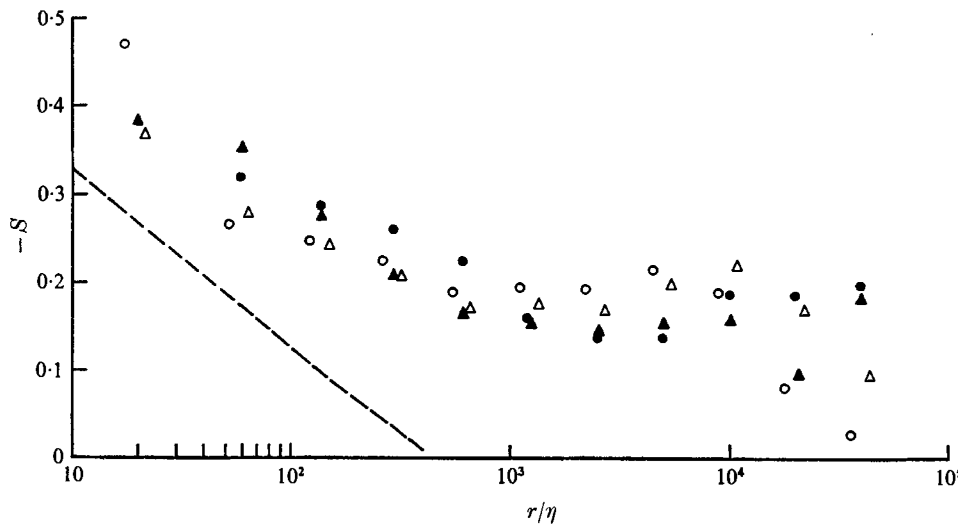

FIGURE 5. Skewness factor as a function of separation distance normalized with Kolmogorov length. The dashed curve is the same quantity for grid-generated turbulence.

in the plateau region is about -0.18, while the values obtained from (4) using the averaged measured C_2 and K' are $S = -0.225$ and -0.17, respectively. Hence, there is only fair agreement between the measured S and that computed from C_2, but the S determined from the unusually large value of K' is very close to the measured value. At larger separations, the widely different behaviour and increasing scatter of the data for different heights may be caused both by the different behaviour of the structure functions for these levels and a lack of sufficiently large samples of data to achieve sufficient statistical convergence.

The striking contrast between the slowly varying behaviour of the skewness factor in atmospheric turbulence and laboratory measurements of the same quantity in relatively low Reynolds number turbulent flows is illustrated in figure 5 by including data for the time-skewness factor in grid turbulence. The behaviour of this function in grid turbulence is well established, as there is excellent agreement between the data of Frenkiel & Klebanoff (1967), and Van Atta & Chen (1968). In grid turbulence, the skewness factor is a monotonically rapidly decreasing function of separation distance which exhibits no trace of a plateau, apparently because the Reynolds number is too small for the existence of an inertial subrange.

4.3. *Fourth-order structure function*

As anticipated from work in grid turbulence, the scatter between individual runs of the measured fourth-order structure functions was much smaller than for the triple moments. As shown in figure 6, the measured values vary smoothly and monotonically up to the largest separations considered. The normalized data collapse fairly well into a single curve, with the exception of one of the runs

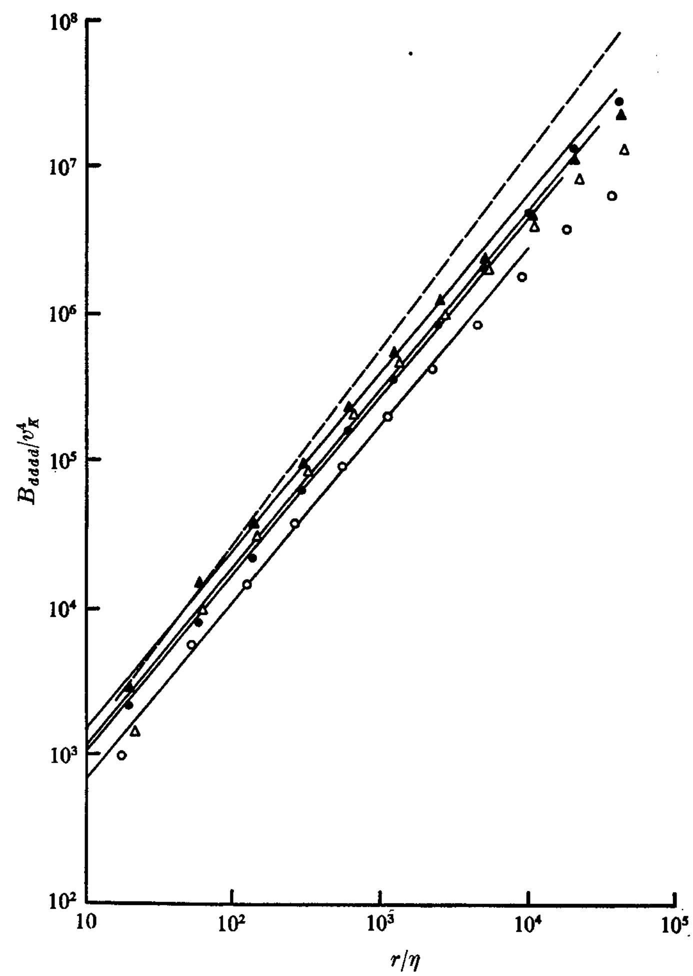

FIGURE 6. Fourth-order structure functions normalized with Kolmogorov scales. The solid lines have a slope of 1·222, while the dashed line has a slope of $\frac{4}{3}$.

at $z = 3$ m, which also produced the lowest values for the other normalized structure functions. The difference in the exponent of r predicted by the original and modified theories is twice as large as for the second-order structure function. As may be seen from figure 6, $B_{dddd}(r)$ is clearly not proportional to $r^{\frac{4}{3}}$ as predicted by the original theory (1), but very closely follows the $r^{1 \cdot 222}$ dependence predicted by the modified theory. Thus, as far as structure functions of velocity are concerned, in order to observe an unambiguously defined departure from the original predictions of Kolmogorov, it is necessary to measure functions of at least fourth order.

The measured flatness factors $F(r)$, shown in figure 7, are smoothly decreasing functions of r, as found earlier by Stewart *et al.* (1970). Because of the previously discussed behaviour of the fourth- and second-order structure functions, the flatness factor decreases nearly like $r^{-0\cdot111}$ over a considerable range for intermediate values of r/η and is roughly proportional to $r^{-0\cdot22}$ over a short range for smaller

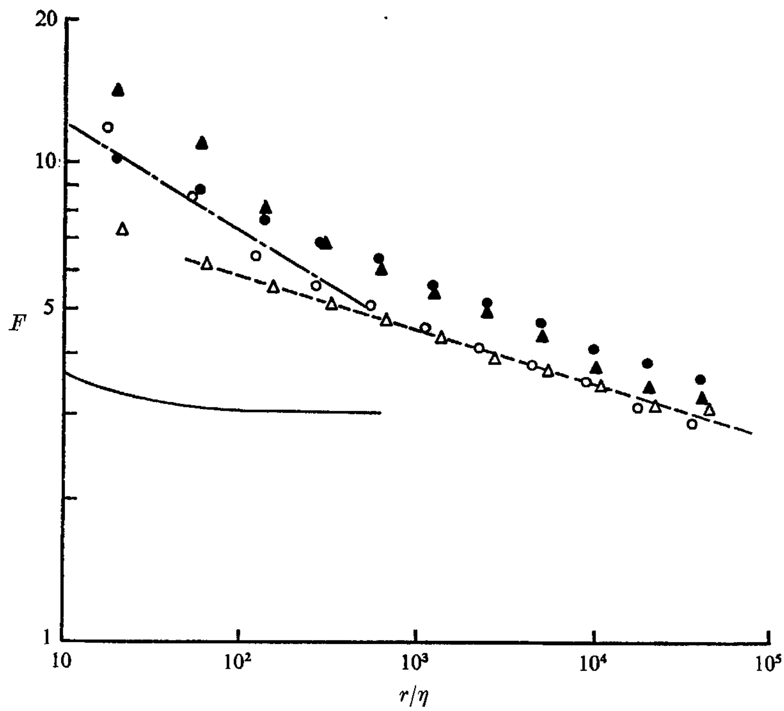

FIGURE 7. Flatness factor as a function of separation distance normalized with Kolmogorov length. – – – –, a slope of $-0\cdot111$; — - — - — -, a slope of $-0\cdot22$. ——, the same quantity for grid-generated turbulence.

values of r/η. For large separations, $F(r)$ approaches a value of 3, which would be obtained for all r if the joint probability density of u and u' were a bivariate Gaussian distribution. The values of F are several times larger than those measured in unsheared grid turbulence for corresponding values of r/η, indicating that the joint probability density is considerably less Gaussian for the present data. This is to be expected, as even the one-dimensional probability densities of u and u' are markedly non-Gaussian in a shear flow. In so far as the flatness factor is a measure of the intermittency of the turbulence, the present measurements confirm the familiar result that the intermittency becomes increasingly stronger as the separation distance or scale of the eddies considered is reduced.

4.4. *Probability distributions of velocity differences and derivatives*

The non-zero values for the third-order structure function and skewness, as well as the fact that the flatness factor is considerably greater than 3·0, are consequences of the fact that the joint probability density of the fluctuating velocities at different times (or points, using Taylor's hypothesis) is non-Gaussian. The joint probability density has not been measured in the present study, but

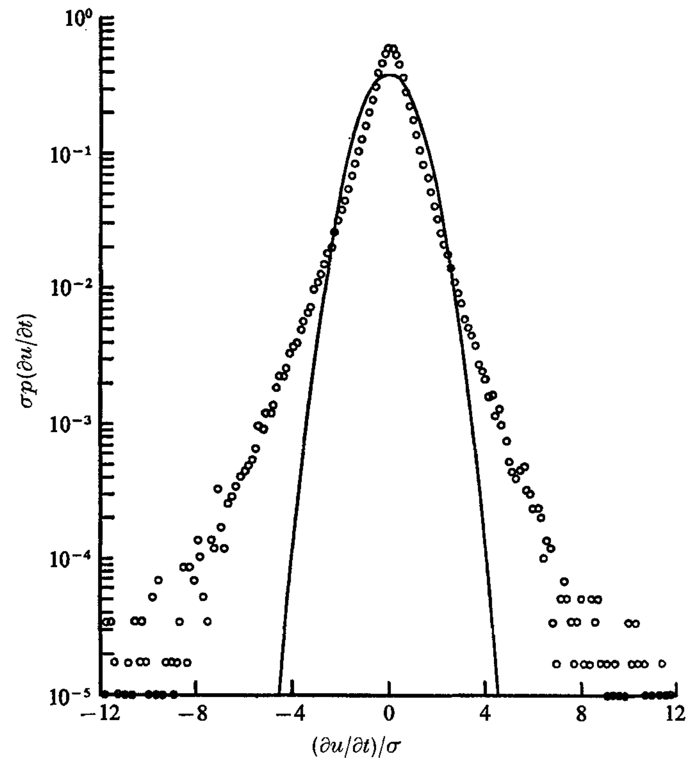

FIGURE 8. Probability density of $\partial u/\partial t$ for $z = 3$ m, $u = 11$ m/s. σ is the standard deviation of $\partial u/\partial t$. ———, the Gaussian distribution.

some probability densities for both the velocity differences and the velocity derivative were determined and found to be very similar to previous atmospheric measurements by other investigators. The probability densities for the velocity difference and the derivative were very similar. As illustrated by the example in figure 8, the probability of very small values of $\partial u/\partial t$ or $u-u'$ and values of $|\partial u/\partial t|$ or $|u-u'|$ greater than 3σ is considerably larger than would be expected for a Gaussian distribution, while the probability of intermediate values is smaller. The data shown are very similar to those obtained by Sheih (1969) from airborne hot-wire measurements.

Yaglom's model of the cascade process predicts that the distribution of $\tilde{\epsilon}$ is lognormal. Assuming local isotropy, one can test for lognormality by measuring $p[(\partial u/\partial x)^2]$, or $p[(u-u')^2]$. The present data for the distribution functions of these quantities, an example of which is shown in figure 9, is very similar to that previously obtained by Gurvich (1966), Sheih (1969), and Gibson, Stegen & Williams (1970). They interpreted the linear portion of the distribution function as evidence of lognormality and ascribe any curvature to the effects of noise, which could significantly increase the probability density for small values of

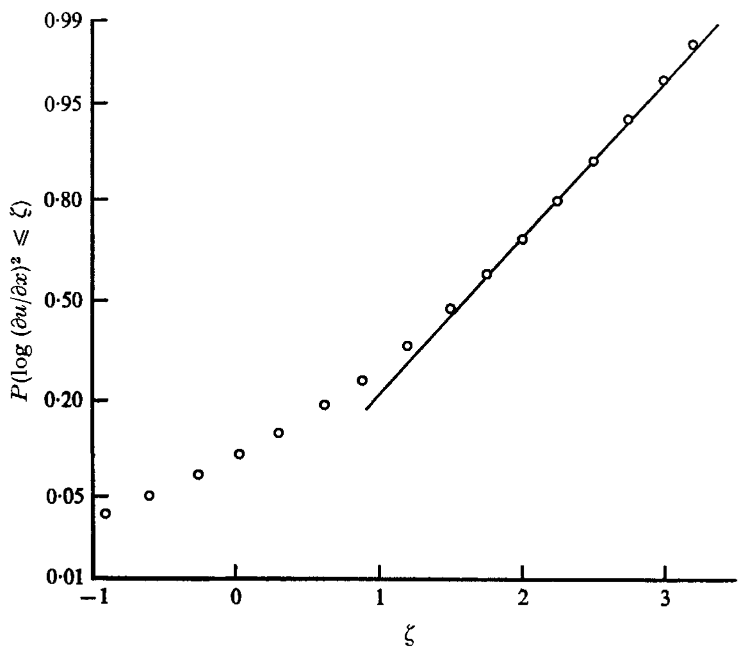

FIGURE 9. Example of measured distribution function of log $(\partial u/\partial x)^2$. If measured values of $(\partial u/\partial x)^2$ were lognormally distributed for all $\partial u/\partial x$, then the entire distribution function would lie on a straight line in this representation.

$\partial u/\partial x$. However, Stewart *et al.* (1970) have found that direct comparisons of the probability densities for similar data show significant deviations from lognormal behaviour at both high and low values. In view of these differences, definite conclusions with regard to lognormality of the present data will require a more complete analysis.

5. Conclusions

Measured fourth-order structure functions of atmospheric turbulence do not vary with separation as predicted by the original Kolmogorov theory, but are consistent with the behaviour predicted by modifications of the theory which attempt to account for the intermittency of energy dissipation. In this respect, no conclusive evidence was found of a measurable effect on the second-order structure function. The measured absolute value of the third-order structure function and its nearly linear dependence on r over a restricted range of r are consistent with Kolmogorov's inertial subrange analysis of the Kármán–Howarth equation. The measured skewness factor exhibits a broad plateau in the same range, with a value near that predicted from C_2, the measured second-order structure function constant.

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